Pearson International GCSE
Further Pure Mathematics 4PM1

Effective Delivery and Assessment

Aims and Objectives

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Course description: This online event is designed for teachers wishing to improve their knowledge of the International GCSE Further Pure Mathematics specification.

Delegates will:
- Explore different approaches to teaching the content of the International GCSE 4PM1 specification
- Understand the assessment process
- Consider candidate responses alongside mark schemes from previous examinations
- Review examiner report to give feedback on recent candidate performance
- Address common issues and FAQs

International GCSE Training

Further Pure Mathematics

Advice to Candidates

- Candidates would be well advised to **state** formulae before using them. (Because of the way the mark scheme is applied)

- Candidates should be very careful in their use/omission of brackets.(example later)

- In ‘show that’ questions **all necessary steps** must be shown.(example later)

- The rubric clearly states that answers without working may not gain full marks.

- A final point is that candidates should be encouraged to work neatly. This will not only help the candidate organise their thoughts, but help the examiner to award marks.

1 Logarithmic functions and indices

What students need to learn

A The functions a^x and $\log_b x$ (where *b* is a natural number greater than one)

B Use and properties of indices and logarithms, including change of base

C Simple manipulation of surds

D Rationalising the denominator

Logarithms

At its very simplest, questions such as

Find the value of

$$\log_3 9$$

could be asked.
(Paper 2 June 2015 Q 10 (a))

This is a 1 mark question as it is expected that candidates are able to 'write down' the correct value.

Logarithms

This is part (b) of the same question

'Given that'

$$\log_9 4 = k \log_3 4$$

Find the value of k (2)

Candidates should be aware that part (a) leads into part (b) and therefore they should be looking to use the result from part (a).

The level of demand is moderate and leads into the final part of the question with a higher level of demand.

(c) 'Show that'

$$2x\log_3 x - 3\log_3 x - 4x\log_9 4 + 6\log_9 4 = \log_3\left(\frac{x}{4}\right)^{(2x-3)}$$

(6)

Again, part (b) leads into part (c).

The instruction 'show that' means that candidates must show EVERY step in their working, and note that there are 6 marks available in this part. This is an indication that a lot of work is required for that many marks.

Logarithms

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Discuss the working that is required to convince the examiner so that a candidate can successfully show that;

$$2x\log_3 x - 3\log_3 x - 4x\log_9 4 + 6\log_9 4 = \log_3\left(\frac{x}{4}\right)^{(2x-3)}$$

Logarithms

The final part of the same question now develops using the word 'hence'. This implies that the previous result is to be used to solve the equation, and

(d) Hence solve the equation

$$2x\log_3 x - 3\log_3 x - 4x\log_9 4 + 6\log_9 4 = 0$$

(3)

.

Logarithms

This means, solve;

$$\log_3\left(\frac{x}{4}\right)^{(2x-3)} = 0$$

Candidates should always check that they find a complete solution. The unknown appears twice in the equation, so candidates should be aware that there could be up to 2 roots.

Logarithms

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From
Paper 1
Jan 2016

10 Given that $2\log_y x + 2\log_x y = 5$

(a) show that $\log_y x = \dfrac{1}{2}$ or $\log_y x = 2$

(5)

(b) Hence, or otherwise, solve the equations

$$xy = 27$$

$$2\log_y x + 2\log_x y = 5$$

(6)

The next slide shows the full extent of the candidates answer to both parts.

Logarithms

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$xy = 27$ $\quad y=3,\ x=9$

$2\log_y x + 2\log_x y = 5$ $\quad$ or $x=3,\ y=9.$

(6)

$2\log_y x + 2\frac{1}{\log_y x} = 5.$

$\log_y x = A.$

$2A + 2\frac{1}{A} = 5.$

$2A^2 + 2 = 5A.$

$2A^2 - 5A + 2 = 0.$

$(2A-1)(A-2) = 0.$

$A = \frac{1}{2}$ or $2.$

$\therefore \log_y x = \frac{1}{2}$ or $\log_y x = 2.$

$\cancel{y = \frac{27}{x}}$ $\quad x = \frac{27}{y}.$

(b) $x = y^2.$

$x = \frac{27}{y}.$ $\quad y^2 = \frac{27}{y}.$

$y^3 = 27.$

$y = 3,\ x = 9.$

This is the full extent of the candidates answer.

What do you think The candidate has missed?

Logarithms

This was marked as follows

M1 – for converting
one logarithm to a power

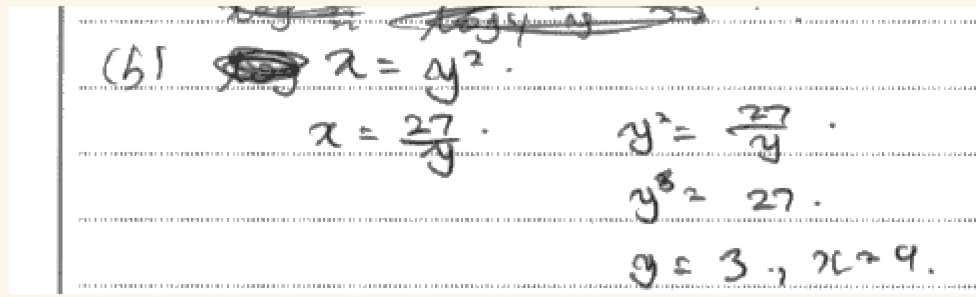
A0 – only one log converted

M1 – for one correct substitution

A1 – one correct pair

M0 – second root from (a) not
converted

A1 – only one correct pair


(b) $x = y^2$.
$x = \frac{27}{y}$.
$y^2 = \frac{27}{y}$.
$y^3 = 27$.
$y = 3$, $x = 9$.

This candidate has lost 2 marks because they did not use the second root that they found from part (a). A needless loss of 2 marks.

Indices

Simple questions could be asked, such as,

Find the exact solution of $4^{(x-2)} = 8^{(3x-1)}$

A key word for candidates here is exact, so the solution will be a rational number or a surd.

The following slides show three possible methods.

1) Discuss the three approaches.
2) Is one method 'better' than the others?
3) Should students be exposed to more than one approach?

Indices

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Method 1

$$4^{(x-2)} = 8^{(3x-1)}$$

$$x-2 = (3x-1)\log_4 8$$

$$x-2 = \frac{3}{2}(3x-1)$$

$$2x-4 = 9x-3$$

$$-1 = 7x$$

$$x = -\frac{1}{7}$$

Indices

Method 2

$$4^{(x-2)} = 8^{(3x-1)}$$

$$2^{2(x-2)} = 2^{3(3x-1)}$$

$$2x-4 = 9x-3$$

$$-1 = 7x$$

$$x = -\frac{1}{7}$$

Indices

$$4^{(x-2)} = 8^{(3x-1)}$$

$$(x-2)\log 4 = (3x-1)\log 8$$

$$\frac{(x-2)}{(3x-1)} = \frac{\log 8}{\log 4}$$

$$\frac{(x-2)}{(3x-1)} = \frac{3}{2}$$ (usually using a calculator)

$$x-2 = \frac{3}{2}(3x-1)$$

$$-1 = 7x$$

$$x = -\frac{1}{7}$$

2 The Quadratic function

A The manipulation of quadratic expressions
B The roots of a quadratic equation
C Functions of the roots of a quadratic equation

The Quadratic Function

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Completing the square of a quadratic function

The questions can be simple, or as in the following example, the level of demand has been increased by including a negative coefficient of x^2 in the function.

$$\mathrm{f}(x) = 4 + 3x - x^2$$

(a) Write $\mathrm{f}(x)$ in the form $P - Q(x+R)^2$

The Quadratic Function

Completing the square of a quadratic function

Candidates are also expected to interpret their answer and the questions goes on to ask:

The curve *C* has equation

$$y = 4 + 3x - x^2$$

(b) Find the coordinates of the maximum point of *C*. (1)

Candidates should look at the marks being awarded to be aware of the amount of work involved. In this case, there is only 1 mark available, so this is a strong hint that the answer can be written down using the result from part (a).

The Quadratic Function

The curve C has equation $y = 4 + 3x - x^2$

(b) Find the coordinates of the maximum point of C.

(b) $\frac{dy}{dx} = 3 - 2x$

put $\frac{dy}{dx} = 0$

so, $3 - 2x = 0$

$\therefore x = 3/2$

put $x = 3/2$ in $y = 4 + 3(3/2) - (3/2)^2$

$= \frac{25}{4}$

Maximum point $= (3/2, \frac{25}{4})$

In this case the candidate has not interpreted their answer from (a), but has differentiated (correctly) to find the coordinates.
This was still only worth one mark

The Quadratic Function

Roots of a quadratic equation

Students are expected to understand and use;

the equation $ax^2 + bx + c = 0$ has roots α and β

such that $\alpha + \beta = -\frac{b}{a}$ $\alpha\beta = \frac{c}{a}$

The most common source of error in this topic is the inability to expand polynomials correctly and this is pre-requisite in this topic.

The Quadratic Function

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Roots of a quadratic equation

We frequently see $(\alpha+\beta)^3 = \alpha^3+\beta^3$ or

even $(\alpha+\beta)^2 = \alpha^2+\beta^2$

Example from Paper 1 2016

$$f(x) = 3x^2 - 5x - 4$$

The roots of the equation $f(x) = 0$ are α and β

(a) Without solving the equation $f(x) = 0$, form an equation, with integer coefficients, which has

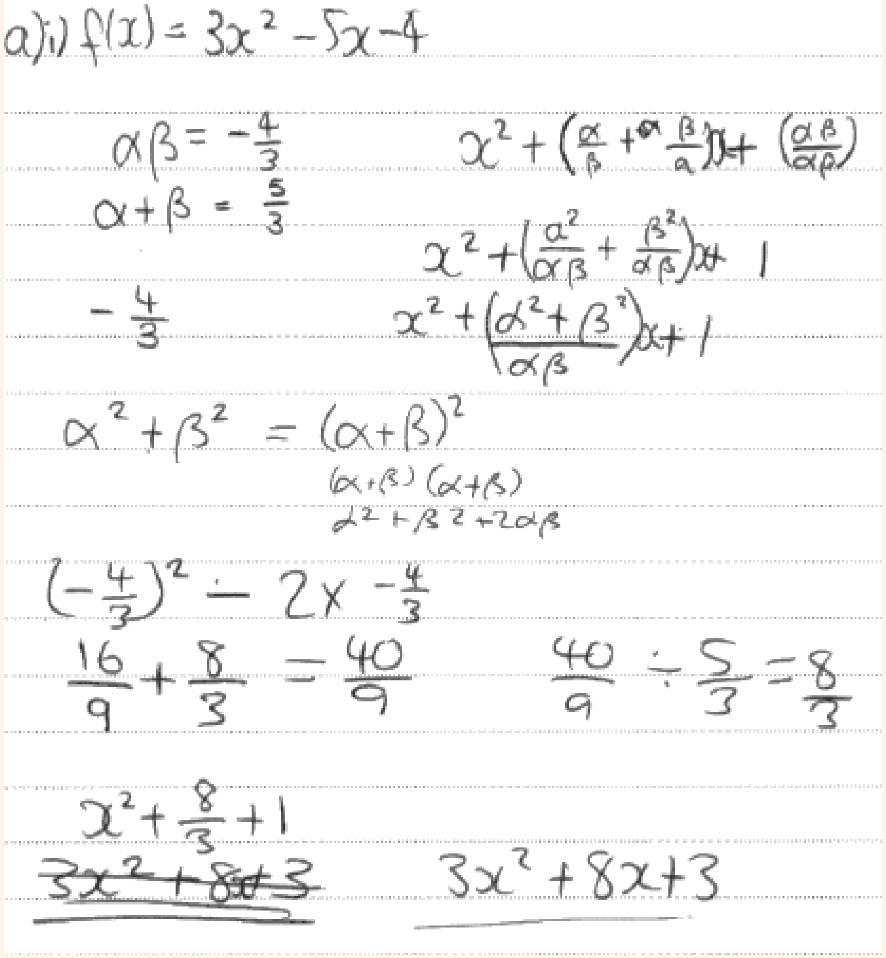
(i) roots $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$

(6)

The Quadratic Function

Roots of a quadratic equation

How do you think this was marked?



a)i) $f(x) = 3x^2 - 5x - 4$

$\alpha\beta = -\frac{4}{3}$

$\alpha + \beta = \frac{5}{3}$

$-\frac{4}{3}$

$x^2 + \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)x + \left(\frac{\alpha\beta}{\alpha\beta}\right)$

$x^2 + \left(\frac{\alpha^2}{\alpha\beta} + \frac{\beta^2}{\alpha\beta}\right)x + 1$

$x^2 + \left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right)x + 1$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2$

$(\alpha+\beta)(\alpha+\beta)$

$\alpha^2 + \beta^2 + 2\alpha\beta$

$\left(-\frac{4}{3}\right)^2 - 2x - \frac{4}{3}$

$\frac{16}{9} + \frac{8}{3} = \frac{40}{9}$

$\frac{40}{9} \div \frac{5}{3} = \frac{8}{3}$

$x^2 + \frac{8}{3} + 1$

$\cancel{3x^2 + 8x + 3}$

$3x^2 + 8x + 3$

The Quadratic Function

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Roots of a quadratic equation

B1 – writes down the sum and product

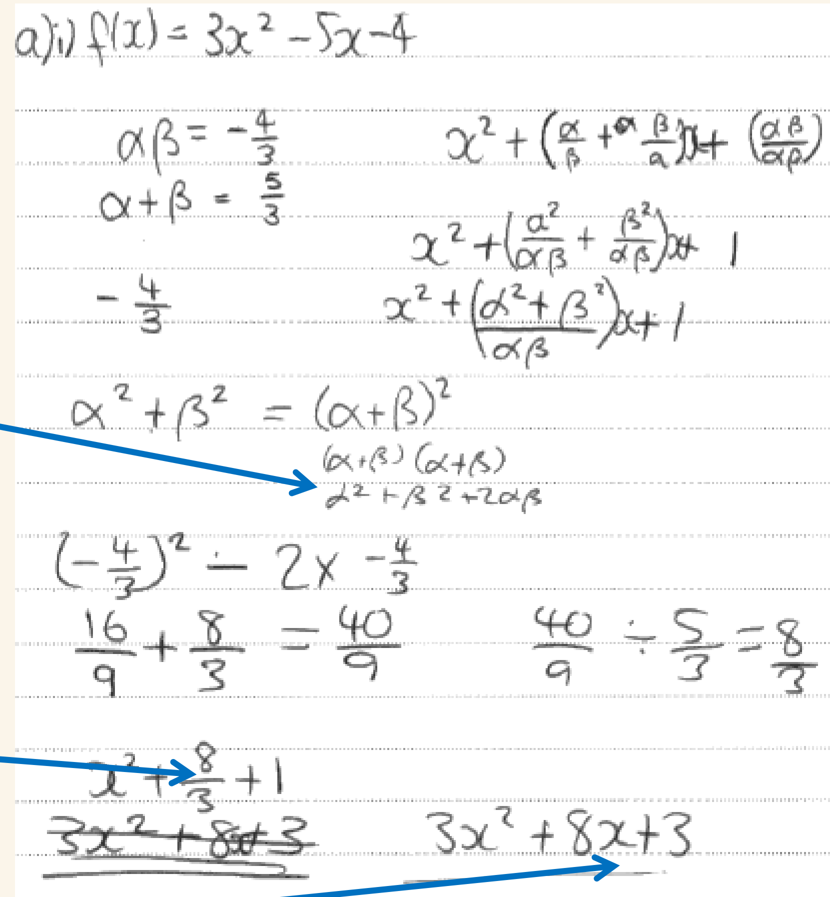
M0 – incorrect algebra for the sum
(despite a correct expansion that has not been used)

A0 – incorrect sum

B1 – for the correct product

M0 – for incorrect use of their erroneous sum
(had it been negative they would have gained this mark)

A0 – incorrect ‘equation’ which is not set = 0



a)i) $f(x) = 3x^2 - 5x - 4$

$\alpha\beta = -\frac{4}{3}$ $\qquad x^2 + \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)x + \left(\frac{\alpha\beta}{\alpha\beta}\right)$

$\alpha + \beta = \frac{5}{3}$ $\qquad x^2 + \left(\frac{\alpha^2}{\alpha\beta} + \frac{\beta^2}{\alpha\beta}\right)x + 1$

$-\frac{4}{3}$ $\qquad x^2 + \left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right)x + 1$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2$

$(\alpha+\beta)(\alpha+\beta)$

$\alpha^2 + \beta^2 + 2\alpha\beta$

$\left(-\frac{4}{3}\right)^2 - 2 \times -\frac{4}{3}$

$\frac{16}{9} + \frac{8}{3} = \frac{40}{9}$ $\qquad \frac{40}{9} \div \frac{5}{3} = \frac{8}{3}$

$x^2 + \frac{8}{3} + 1$

$\cancel{3x^2 + 8x + 3}$ $\qquad 3x^2 + 8x + 3$

The Quadratic Function

This is a quote from the principal examiners report on the above question.

'Most candidates knew what to do here and went about the task well, but some careless calculations and/or algebra defeated a few. The most common error was to give $(\alpha+\beta)^2=\alpha^2+\beta^2$. Others missed out the = 0, for their final equations in this part as well as part (b), but were only penalised once for this error in the question as a whole'.

Marking exercise 1

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Work through question 9 of June 2016

9

$$f(x) = 2x^3 + ax^2 + bx + 15 \qquad \text{where } a \text{ and } b \text{ are constants.}$$

The remainder when $f(x)$ is divided by $(x - 1)$ is -12

The remainder when $f(x)$ is divided by $(x + 1)$ is 48

(a) Find the value of a and the value of b.

(6)

(b) Show that $f\left(\frac{1}{2}\right) = 0$

(1)

(c) Express $f(x)$ as a product of linear factors.

(4)

(d) Solve the equation $f(x) = 0$

(1)

Then use the outline mark scheme to mark the 3 student attempts at question 9.

3 Identities and inequalities

What students need to learn

A Simple algebraic division
B Factor and remainder theorems
C Solution of equations including simultaneous equations
D Simple inequalities
E Graphical representation of linear inequalities.

Key teaching points

The majority of errors in these questions do not come from an inability to identify the techniques required and apply them, but mainly from inaccurate algebra and simple and very surprising errors.

This is an example of what is too frequently seen;

$$(x+3)(x-1)=4$$

$$\Rightarrow (x+3)=4 \text{ or } (x-1)=4$$

$$\Rightarrow x=1,\ 5$$

Identities and inequalities

This question comes from January 2016

Solving simultaneous equations of one linear and one quadratic in 2 variables.

3 Solve the equations

$$3y = 12 - 4x$$

$$(x + 1)^2 + (y - 2)^2 = 4$$

(7)

It will be useful to refer to the mark scheme to this question which is given on page 2 of the Delegate Booklet 2 (Mark Schemes)

Identities and inequalities

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And here is a quote from the Principal Examiners report for this question.

Question 3:

Forming and substituting an equation with y as the subject was the most popular choice and this usually resulted in candidates earning at least the first three marks. A variety of algebraic errors were seen including failing to divide both terms by 3 (or 4) when making y (or x) the subject, errors on substitution such as the +1 or -2 being omitted, poor expansion of the bracketed terms and careless collection of like terms after expansion. Each of these errors meant the A mark for a correct 3-term quadratic was lost along with the final two A marks for the correct values of x and y.

And the report goes on to add

Any prior algebraic errors resulted an incorrect quadratic which was usually dealt with by using the quadratic formula. This meant that candidates still had a chance to earn the final M mark for solving their quadratic with an appropriate method. However, there were some incorrect expressions for the quadratic formula or errors on substitution without a correct formula which mean the M mark was also lost.

Identities and inequalities

Here is an example of a candidate ‘solving’ their **correctly found** quadratic equation.

The correct solution of this quadratic is indeed $x = \frac{3}{5}$

but because the method is incorrect and x = 0 is **not** a root, then the 2 available marks were lost here.

Neither did the candidate go on to find a value for *y*, and so lost the final mark as well, and having completed the ‘difficult’ part of the question correctly achieving only 4/7 available marks.

$$\frac{25}{9}x^2 - \frac{10}{3}x + 1 = 0$$

$$x(5x-3) = 0$$

$$x = 0 \qquad 5x - 3 = 0$$

$$x = \frac{3}{5}$$

4 Graphs

What students need to learn

A Graphs of polynomials and rational functions with linear denominators

B The solution of equations and transcendental functions by graphical methods

A Graphs of polynomials and rational functions with linear denominators

These questions nearly always involve graphs requiring candidates to find equations of asymptotes.

A significant number of candidates have little understanding of asymptotes and how their equations could be determined.

A Graphs of polynomials and rational functions with linear denominators

Question 9 June 2015

9 A curve C has equation $y = \dfrac{3x+1}{2x+3} \qquad x \neq -\dfrac{3}{2}$

(a) Write down an equation of the asymptote of C which is parallel to

(i) the x-axis,

(ii) the y-axis.

(2)

The instruction given is ‘write down’

How could this be taught so that a candidate can spot the asymptotes immediately and ‘write’ them down correctly?

Graphs

B The solution of equations by graphical methods

These questions generally follow a similar format and candidates are expected to;

- complete a table of values by calculating values to a specified degree of accuracy.
- draw the curve of the function.
- solve an equation which involves some manipulation of the given equation in order to be able to draw a straight line.

Graphs – solution of equations

Paper 1 2016 Q7

This is a candidate's response to part (a). One could imagine that the value they find for *x* = 0.5 is a slip.

7 (a) Complete the table of values for $y = 2^x - 4$, giving your answers to 2 decimal places.

$x = 2.8$
$y = 7$

x	0	0.5	1	1.5	2	2.5	2.75	3
y	−3	2.59	−2	−1.17	0	~~[illegible]~~	2.73	4

1.67

(2)

The next slide shows the graph the student drew for their values.

Graphs – solution of equations

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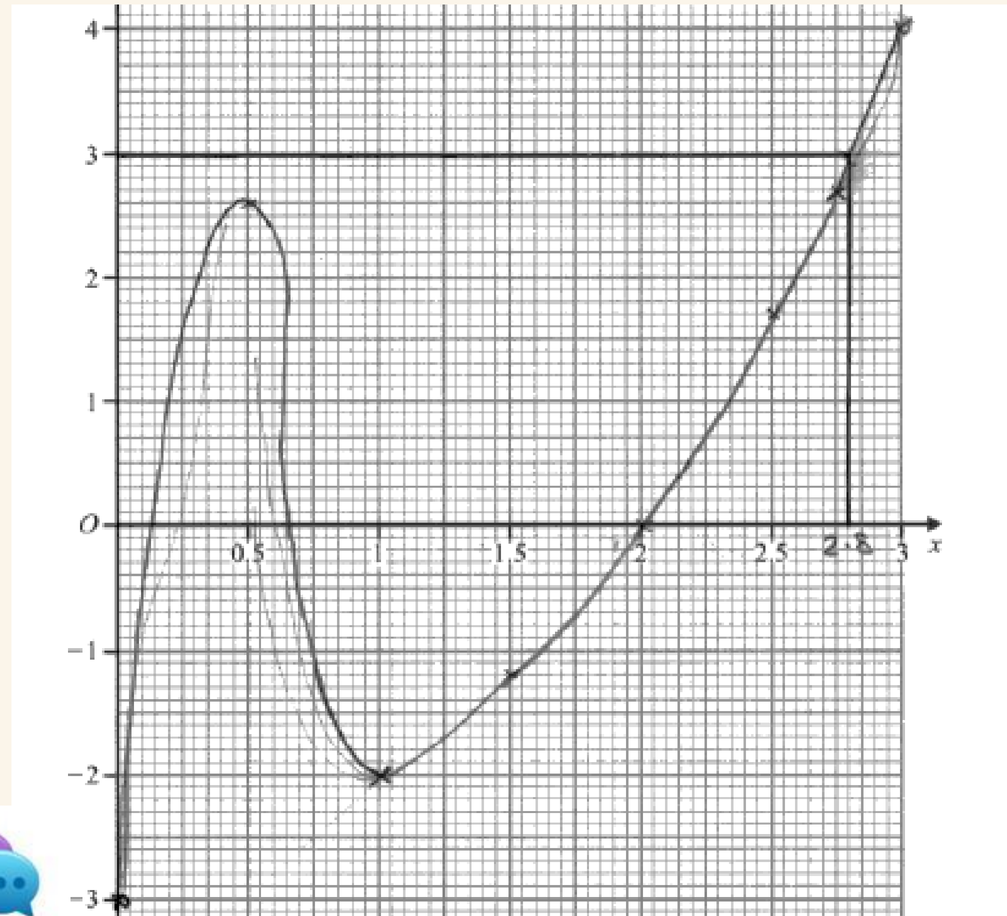
Paper 1 2016 Q7

(b) Drawing the graph

The specification states ‘A knowledge of the shape of the graphs of log *x* is expected’. The candidate clearly did not!

They have carried forward their error in part (a) because the value for *y* when *x* = 0.5 should have obviously been −2.59.

How was this marked?



Graphs – solution of equations

Paper 1 2016 Q7

7 (a) Complete the table of values for $y = 2^x - 4$, giving your answers to 2 decimal places.

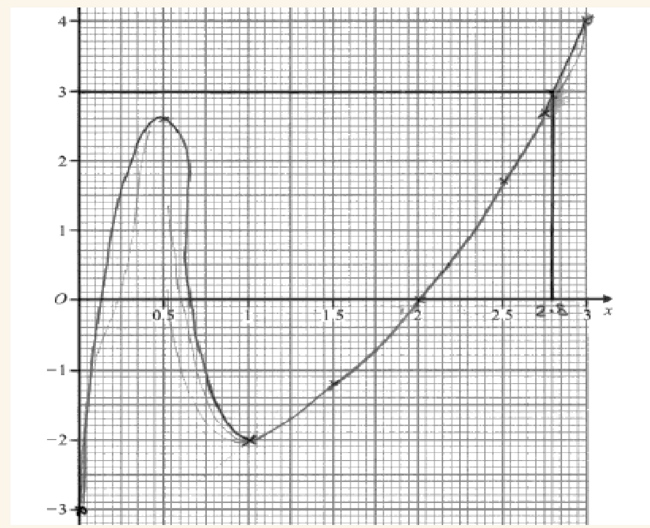
x	0	0.5	1	1.5	2	2.5	2.75	3
y	−3	2.59	−2	−1.17	0	1.67	2.73	4

This was marked as follows;

(a) Table

B1 – for one value correct (there are two correct values)

B0 – one incorrect value



(b) Graph

B1ft – all points plotted within half of a square

B1ft – all points joined up in a smooth curve

The candidate has achieved marks in part (b) because they have understood the question.

Graphs – solution of equations

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Paper 1 2016

The level of demand now increases, and there is a clear instruction given.

(c) Use your graph to obtain an estimate, to one decimal place, of the value of $\log_2 7$
Show clearly how you used the graph.

(3)

Here are two examples of how far too many candidates answered this part of the question.

c)* $\log_2 7 = 2.81$

(c) $\log_2 7 = 2.807354922$
$= 2.81$
Value of $\log_2 7 = 2.8$

Graphs – solution of equations

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Paper 1 2016 Q7

Part (c)

(c) Use your graph to obtain an estimate, to one decimal place, of the value of $\log_2 7$
Show clearly how you used the graph.

(3)

How would you show clearly how you used the graph?

Graphs – solution of equations

Paper 1 2016 Q7

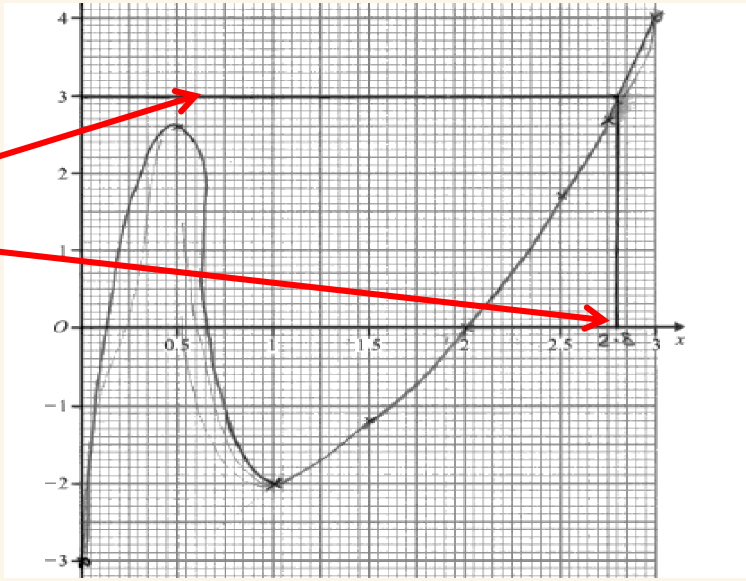
Two possible methods, but both **MUST** use logs.

Method 1

$$\log_2 7 = x \Rightarrow 2^x = 7 \Rightarrow 2^x - 4 = 3 \text{ and } y = 2^x - 4$$

Draw the line y = 3 and find the corresponding value of x.

Despite the incorrect graph the line is drawn correctly.



Graphs – solution of equations

Paper 1 2016 Q7

Two possible methods, but both **MUST** use logs.

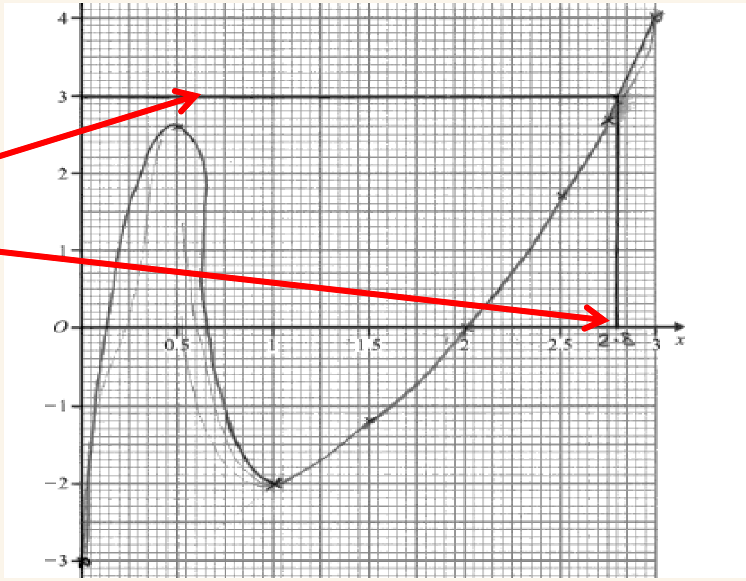
Method 2

$$2^x - 4 = y \Rightarrow 2^x = y + 4 \Rightarrow x = \log_2(y+4)$$

$$\Rightarrow y + 4 = 7 \Rightarrow y = 3$$

As before, draw the line *y* = 3 and find the corresponding value of *x*.

Despite the incorrect graph the line is drawn correctly.



5 Series

What students need to learn

A Use of Σ notation

B Arithmetic and geometric series

Series

In the new specification 4PM1, the formulae will now be given for the sum of an arithmetic and geometric series, and also the sum to infinity of a geometric series.

The formulae for the *n*th terms however, will not be given.

The questions set in this specification tend not to be routine, but require some thought before formulae are applied almost in an ‘automatic pilot’ mode.

The question in the following slide is an example.

Series

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An arithmetic series has first term p and common difference p where $p \neq 0$
A geometric series also has first term p. The common ratio of this geometric series is r.
The sum of the first three terms of the arithmetic series is equal to the sum of the first three terms of the geometric series.

Given that $r > 0$

show that $r = \dfrac{-1+\sqrt{21}}{2}$

(5)

A good start in questions such as these is to simply write out the terms as described in the question, and then it becomes obvious how to proceed.

B1 $p+(p+p)+(p+2p) = p+pr+pr^2$ B1

$$6p = p(1+r+r^2)$$

$$6 = 1+r+r^2 \Rightarrow r^2+r-5=0$$ M1

(Use quadratic formula) $r = \dfrac{-1+\sqrt{1^2-4\times1\times-5}}{2} = \dfrac{-1+\sqrt{21}}{2}$ M1A1

The following slide is an example of how most candidates attempted this question

Series

The work is all correct so far!

However, over-complicating the question has resulted in a cubic, which the candidate did not attempt to solve, (although more than a few candidates attempted to solve a cubic using the quadratic formula!)

This was worth only 3 marks in total.

$S_3 = \frac{1}{2} \times 3 [2 \times P + (3-1)P]$ arithmetic series

$S_3 = \frac{P(1-r^3)}{1-r}$ geometric series

B1

$\frac{1}{2} \times 3 [2 \times P + (3-1)P] = \frac{P(1-r^3)}{1-r}$ B1

$\frac{3}{2}(2P + 2P) = \frac{P(1-r^3)}{1-r}$

$6P = \frac{P(1-r^3)}{1-r}$

$6P(1-r) = P(1-r^3)$

$6 - 6r = 1 - r^3$

$r^3 - 6r + 5 = 0$ M1

Specification International GCSE 4PM1

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6 The binomial series

What students need to learn

Use of the series when:

I. n is a positive integer

II. n is rational and $|x| < 1$

The validity condition for (*II*) is expected

Binomial series for any rational *n*.

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There are three problem areas in this topic.

1. Poor bracketing in expansions. This is particularly so in questions such as;

Expand $\left(1+3x^2\right)^{-\frac{1}{3}}$, $3x^2 < 1$, in ascending powers of x, up to and including the term in x^6, simplifying each term as far as possible.

2. Students have difficulties dealing with examples such as

$$\frac{1}{\sqrt{4-x}}$$

3. Considering that many candidates have sophisticated calculators, the number of arithmetical errors is alarming.

Binomial series for any rational *n*

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Here is an example from January 2016 where the candidate has failed to answer the question correctly because of poor bracketing. In general, this part of the question was not well attempted by most.

Given that $\dfrac{1}{\sqrt{4-x}}$ can be written as $p(1-qx)^{-\frac{1}{2}}$

(a) find the value of p and the value of q.

(2)

$\dfrac{1}{\sqrt{4-n}}$

$= 1 \times (4-n)^{-1/2}$

$= 4\left(1-\frac{1}{4}n\right)^{-1/2}$ (A)

Had the candidate used brackets correctly, they would have not made this error

Binomial series for any rational *n*

Here is another example of poor bracketing.
The level of demand in the question has increased to,

Given that the first three terms of the expansion of $\dfrac{2(1+x)}{\sqrt{4-x}}$ are $a + bx + cx^2$

(c) find the exact value of

(i) a (ii) b (iii) c

(3)

(c) $\dfrac{2(1+x)}{\sqrt{4-x}}$

$= 2\cancel{x}(1+x) \times (4-x)^{-\frac{1}{2}}$

$(2+2x) \times \frac{1}{2} + \frac{1}{4}x + \frac{1}{16}x^{\frac{1}{2}} + \frac{1}{128}x^{\frac{3}{2}}$

$= 1 + \frac{1}{2}x + \frac{1}{8}x^{\frac{1}{2}} + \frac{1}{64}x^{\frac{3}{2}} + x + ([illegible]) \frac{1}{2}x^2 + \frac{1}{8}x^{\frac{3}{2}} + \frac{1}{64}x^{\frac{5}{2}}$

The candidate has used their incorrect expansion from part (b) correctly, and has in fact multiplied out their 'invisible' bracket correctly to gain both M marks.

However, this is very risky and in most cases leads to errors.

Binomial series for any rational *n*.

An **enrichment** suggestion

Students could start by finding the (interesting) expansions of

$$\frac{1}{1-x} = (1-x)^{-1}$$

and

$$\frac{1}{(1-x)^2} = (1-x)^{-2}$$

and then extend to $\frac{1}{(a \pm x)^2}$ and $\frac{1}{(a \pm bx)}$ etc

Specification International GCSE 4PM1

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7 Scalar and vector quantities

What students need to learn

A The addition and subtraction of coplanar vectors and the multiplication of a vector by a scalar

B Components and resolved parts of a vector

C Magnitude of a vector

D Position vector

E Unit vector

D Use of vectors to establish simple properties of geometrical figures

Scalar and vector quantities

Vector questions always clearly differentiate between the strongest and the weakest candidates.

Many marks in questions in vectors are lost due to two reasons;

1. candidates do not have a consistent approach to directions which are so crucial in this work, and
2. candidates use poor notation.

Questions frequently begin with fairly routine demands. The next slide gives a typical example.

Vectors

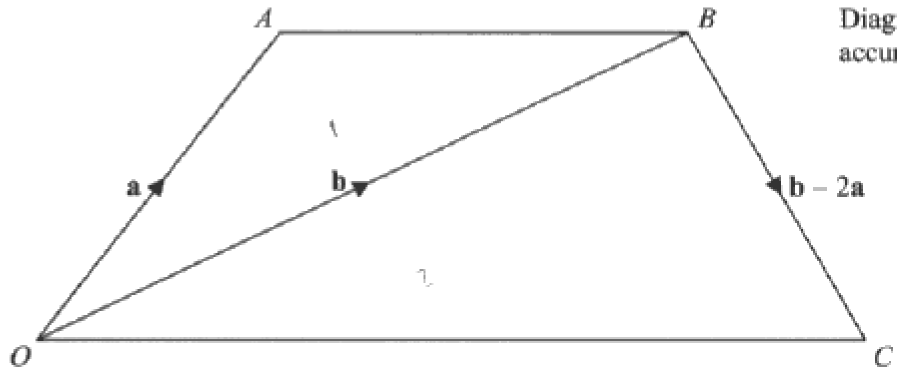

Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a quadrilateral *OABC*

$$\overrightarrow{OA} = \mathbf{a},\ \overrightarrow{OB} = \mathbf{b} \text{ and } \overrightarrow{BC} = \mathbf{b} - 2\mathbf{a}$$

(a) (i) Prove that $\overrightarrow{AB}$ is parallel to $\overrightarrow{OC}$

(ii) Show that $AB : OC = 1:2$

It is always advisable to establish correct vector statements **first.** This will help the candidate AND crucially score the Method mark.

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} \Rightarrow -\mathbf{a} + \mathbf{b}$$

$$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} \Rightarrow \mathbf{b} + \mathbf{b} - 2\mathbf{a} = 2\left(-\mathbf{a} + \mathbf{b}\right)$$

In a proof, a conclusion is required;

$\overrightarrow{OC} = 2\overrightarrow{AB}$ so same direction and ratio is

$$\overrightarrow{AB} : \overrightarrow{OC} = 1:2$$

Vectors

The second part of the question increases the level of demand and it continues with;

The point D lies on OB such that $OD:DB = 2:3$

(b) Find the ratio of the area of $\triangle ODC$: the area of $\triangle OAB$.

(6)

These parts of vector questions are usually amongst the most challenging in the whole paper, but they need not be so.

The most common error in vector questions involving length or area is to attempt to use vectors instead of lengths.

In this case, the solution is straightforward when areas of triangles are used;

either $\frac{1}{2}ab\sin C$ or half x base x height.

Discuss how you could attempt this question.

8 Rectangular Cartesian coordinates

What students need to learn

A The distance between two points
B The point dividing a line in a given ratio
C Gradient of a straight line joining two points
D The straight line and its equation
E The condition for two lines to be parallel or perpendicular

Rectangular Cartesian coordinates

These questions are always generally well attempted.

Key points which candidates would do well to note.

1. Always draw a sketch. A common feature of poor attempts at these questions is the lack of a good careful sketch.
2. Dividing a line in a given ratio – those candidates who attempt to use the formula usually make mistakes. Those who use similar triangles, nearly always get it right.

Rectangular Cartesian coordinates

And finally:

Many candidates do not read questions carefully and find the equation of the normal when the tangent is required, and vice versa.

Please encourage your students to read these questions very carefully to avoid losing quite unnecessary marks.

9 Calculus

What students need to learn

A Differentiation and integration of sums of multiples of powers of x

B Differentiation of a product, quotient and simple cases of a function of a function

C Applications to simple linear kinematics and to determination of areas and volumes

D Stationary points and turning points

E Maxima and minima

F The equations of tangents and normals to the curve

G Application of calculus to rates of change and connected rates of change

Differentiation

Be efficient when using the **chain rule**

Discourage students from using the **product rule** when the **quotient rule** is more appropriate.

e.g.

$$y = \frac{x^2}{x^3+1} \qquad \Rightarrow y' = \frac{2x(x^3+1) - x^2 \times 3x^2}{(x^3+1)^2} = \frac{2x - x^4}{(x^3+1)^2}$$

As against

$$y = \frac{x^2}{x^3+1} \qquad \Rightarrow y = x^2(x^3+1)^{-1} \Rightarrow y' = 2x(x^3+1)^{-1} + x^2(-1)(3x^2)(x^3+1)^{-2}$$

Marking Exercise 2

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Work through Q 4 shown below from Paper 2 January 2016

4 Given that $y = e^{2x}\sqrt{x+1}$

show that $\frac{dy}{dx} = \frac{e^{2x}(4x+5)}{2\sqrt{x+1}}$

Mark the three student responses given using the outline mark scheme. Discuss how much 'fudging' is present to achieve the final answer.

Differentiation – connected rates of change

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Key result is an application of the chain rule $\dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}x} \times \dfrac{\mathrm{d}x}{\mathrm{d}t}$

which may have to be manipulated in some way.

The following slide has an example of a typical straightforward question.

1) Work through the question first.
2) The first slide shows the standard approach.
3) The second slide shows a more sophisticated approach. Do you think we can/should stretch the more able by teaching this way as well?

Differentiation – connected rates of change

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Question from January 2016

3 The volume, V cm^3, of a sphere of radius r cm is increasing at the rate of 60 cm^3/s.

Find the rate of increase of the radius, in cm/s correct to 2 significant figures, when the volume is 36000π cm^3.

(7)

Differentiation – connected rates of change

Standard method

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 60$$

$$V = \frac{4}{3}\pi r^3 \Rightarrow 36000\pi = \frac{4}{3}\pi r^3 \Rightarrow r = 30$$

$$\frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \times \frac{\mathrm{d}r}{\mathrm{d}V}$$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = 60 \times \frac{1}{4\pi \times 30^2} = 0.0053 \quad (\mathrm{m/s})$$

Differentiation – connected rates of change

Using implicit differentiation (which is beyond the scope of this specification, though occasionally seen)

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 60$$

$$V = \frac{4}{3}\pi r^3 \Rightarrow 36000\pi = \frac{4}{3}\pi r^3 \Rightarrow r = 30$$

Differentiates implicitly

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 4\pi r^2 \times \frac{\mathrm{d}r}{\mathrm{d}t}$$

$$\Rightarrow \frac{\mathrm{d}r}{\mathrm{d}t} = 60 \times \frac{1}{4\pi \times 30^2} = 0.0053 \quad \text{(m/s)}$$

Integration

Key points- the constant of integration

•Don't forget to include +*c* in indefinite integrals. This could result in the loss of just a single mark in a question such as.

$$\mathrm{f}(x) = 3x^2 + \frac{5}{x} + 3 \quad \text{find} \int \mathrm{f}(x)\,\mathrm{d}x$$

The level of demand could be increased by asking candidates to find the equation of a curve given a pair of coordinates, in which case several marks would be lost by this omission.

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Key points - draw a sketch to find an area or a volume

Some question include a sketch and some do not as in the following example.

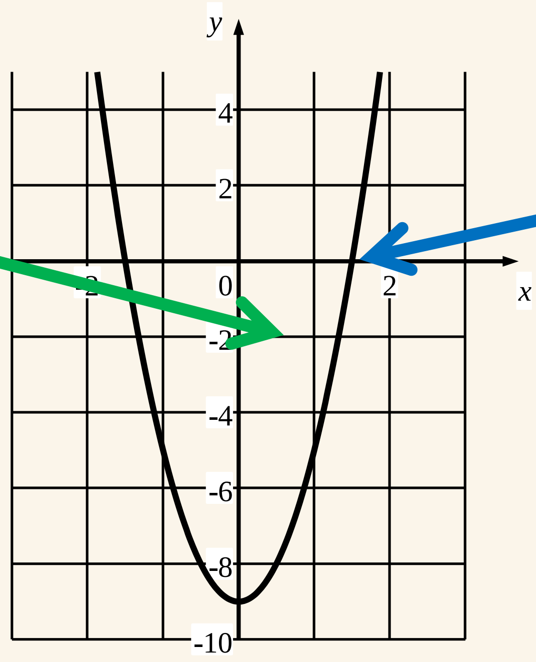
The region enclosed by the curve with equation $y = 4x^2 - 9$, the positive x-axis and the negative y-axis is rotated through 360° about the x-axis.

Use algebraic integration to find, to 3 significant figures, the volume of the solid generated.

Integration

The question does not give any limits, so a sketch would quickly establish what is required to be found.

This is the required region to be rotated.



We need this intercept
With the *x* - axis

Many candidates did not draw the sketch and could not answer the question

Integration

This question is testing volumes of revolution, so a method mark for increasing the power of a variable by 1 was not available. The method marks were dependent on candidates using the correct formula and method for determining the volume.

Step 1 – find the *x* intercepts. $4x^2 - 9 = 0 \Rightarrow x = \pm\frac{3}{2}$ M1A1

Step 2 – establish limits of integration (from sketch) 3/2 and 0

Step 3 – set up integration $\text{Vol} = \pi \int_0^{\frac{3}{2}} \left(4x^2 - 9\right)^2 \, \mathrm{d}x$ M1

Step 4 – integrate $V = \pi \int_0^{\frac{3}{2}} 16x^4 - 72x^2 + 81 \, \mathrm{d}x = \pi \left[\frac{16}{5}x^5 - 24x^3 + 81x \right]$ M1d

Step 5 – evaluate $V = \frac{324\pi}{5} = 203.57... \approx 204 \ \text{(3sf)}$ A1 (5)

Integration

This was the PE report on this question.

Question 1

'This question proved be a difficult beginning for quite a few candidates, as many did not know the correct formula for the volume of revolution, and as the second method mark was dependent on the first, a significant majority of the entry achieved only the first mark for finding $x = \pm\frac{3}{2}$.

Even when candidates knew the formula, many put in the lower limit as $-\frac{3}{2}$ leading to the incorrect answer. In this question, a simple sketch of a correctly placed curve of the quadratic, with the correct area to be rotated shaded (given clearly in the question), would have been of great use in identifying the required region'.

Specification International GCSE 4PM1

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10 Trigonometry

What students need to learn

A Radian measure, including use for arc length and area of sector

B The three basic trigonometrical ratios of angles of any magnitude (in degrees or radians) and their graphs

C Applications to simple problems in two or three dimensions (including angles between a line and a plane and between two planes

D Use of the sine and cosine formulae

E The identity $\cos^2\theta + \sin^2\theta = 1$

F The use of the basic addition formulae of trigonometry

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Drawing a sketch

Not every question includes a sketch, yet more than a few candidates attempt to answer a question without drawing a sketch. This should be a must in any question involving a shape, and candidates should be encouraged to annotate their sketch as they proceed through the question.

For example, no sketch was given in this question.

3 A right pyramid $ABCDE$ has a square base $ABCD$ of side 10 cm.
The height of the pyramid is 8 cm.

(a) Find, to 3 significant figures, the length of AE.

(3)

(b) Find, in degrees to the nearest degree, the size of the angle between the plane ABE and the base $ABCD$.

(3)

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Radians and degrees

The specification can ask for working in radians or degrees. Many candidates seem to be wary of working purely in radians and often work in degrees at first and either leave their answers in degrees (ignoring radians completely), attempt to convert their degrees to radians at the end, or mix up degrees and radians.

Giving your solutions to 3 decimal places, solve the equation

(a) $\cos x = 0.4 \qquad -\pi < x < \pi$ **(2)**

(b) $\tan\left(2\theta + \frac{\pi}{4}\right) = 1.5 \qquad 0 < \theta < \pi$ **(4)**

What has happened here?

b) $\tan\left(2\theta + \frac{\pi}{4}\right) = 1.5$

$2\theta + \frac{\pi}{4} = \tan^{-1}(1.5)$

$2\theta = \tan^{-1}(1.5) - \frac{\pi}{4}$.

$2\theta = 0.19, 180.19$

$\theta = 0.099, 90.099$

Trigonometry

Giving your solutions to 3 decimal places, solve the equation

(a) $\cos x = 0.4$ $\qquad -\pi < x < \pi$ **(2)**

(b) $\tan\left(2\theta + \frac{\pi}{4}\right) = 1.5$ $\qquad 0 < \theta < \pi$ **(4)**

The candidate starts correctly, with the correct statement in lines 2 and 3.

They have found one value for 2*θ*, but instead of finding two values for $\tan^{-1}(1.5)$

they find one, subtract $\frac{\pi}{4}$

and add 180º to find the next value. Two errors!

b) $\tan\left(2\theta + \frac{\pi}{4}\right) = 1.5$

$2\theta + \frac{\pi}{4} = \tan^{-1}(1.5)$

$2\theta = \tan^{-1}(1.5) - \frac{\pi}{4}$

$2\theta = 0.19 , 180.19$

$\theta = 0.099 , 90.099$

Trigonometry

Accuracy Far too many candidates round prematurely leading to erroneous answers. The policy in this specification is strict.

If the answer required is 32.6º, and the answer is given is 32.58º, then this would gain the M mark but lose the A mark.

If however, the answer given was 32.5º, then it loses both marks because it is incorrect.

Trigonometry

Equations with multiple angles

e.g $\sin(3x+60)^{\circ} = \frac{1}{2}$ for $0^{\circ} < x < 360^{\circ}$

What can happen

$$3x + 60 = \sin^{-1}\left(\frac{1}{2}\right) = 30^{\circ}$$

$$\text{So} \quad 3x = -30^{\circ}$$

$$x = -10^{\circ}, 350^{\circ}$$

etc

How should this topic be taught?

Proofs

- Show **each step** to convince the examiner that there is no 'fudging'
- Start with the more complicated side
- Only work on one side at a time

Trigonometry

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A proof or 'show that' is often followed by a trig equation. Some students do not attempt to solve the equation if they cannot do the proof and so lose marks.

In this question, June 2016, the level of demand increases significantly as the question progresses.

However, a proof is given partly to give a candidate who cannot complete the proof, the opportunity of using a given result to attempt another part.

(a) show that $\cos 2\theta = 2\cos^2\theta - 1$ (3)

(b) find a simplified expression for $\sin 2\theta$ in terms of $\sin\theta$ and $\cos\theta$ (1)

(c) show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ (4)

Hence, or otherwise,

(d) solve, for $0 \leqslant \theta < \pi$ giving your answers in terms of π, the equation

$$6\cos\theta - 8\cos^3\theta + 1 = 0$$

(4)

(e) find

(i) $\int\left(8\cos^3\theta + 4\sin\theta\right)\mathrm{d}\theta$

(ii) the exact value of $\int_0^{\frac{\pi}{3}}\left(8\cos^3\theta + 4\sin\theta\right)\mathrm{d}\theta$ (4)

Question/task design

One aspect of question design is DEMAND.

High Demand tasks generally

- are based on less familiar ideas or topics in the specification
- require fluency in knowledge of techniques
- require the ability to have very good manipulative ability in algebra, trigonometry and calculus
- may require solvers to produce their own representation.
- have little apparent structure to help the solver.

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Delegate task

With the two questions given, change them to alter the demand.
How would your change(s) affect the total mark?
How would your changes affect the detailed mark scheme?

Thank you for attending this Edexcel Training Event.

We hope that you have found today's training beneficial and request that you kindly complete our anonymous online evaluation form, which you will receive by email.